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NOMINAL RIGIDITIES AND THE OPTIMAL RATE OF INFLATION

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Abstract

Inflation may enhance the efficiency of the price system in the presence of nominal rigidities. For the price system to function efficiently there is a need for nominal prices to adjust both to real and nominal shocks for relative prices to disseminate the appropriate signals. Since the incentive for price setters to change prices depends not only on the costs of changing prices but also on the realized shocks, it follows that the rate of inflation may affect the incentive to change prices. The higher the rate of inflation the larger the incentive to change prices, and in the presence of real shocks requiring adjustment of relative prices this may lead to a better functioning price system. Empirical evidence supports that nominal rigidities are more prevalent at low rates of inflation. It follows that there can be welfare costs of targeting inflation at too low a level.

Keywords:Nominal rigidities, allocative efficiency, optimal rate of inflationJEL classification:E20, E30

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1. Introduction

It is commonly agreed today that monetary policy should aim at low and stable inflation. Many countries have introduced explicit inflation targeting procedures (see Svensson (1999)) or have taken steps to institutionalize a policy of price stability, cf the institutional set-up of the European Central Bank¹. But what level of inflation should be targeted ? Is the optimal rate of inflation zero or are there strong arguments that a slightly positive rate of inflation - say 2-3% - is optimal? Is it problematic if the policy objective is asymmetric with an upper but no lower bound on the rate of inflation?

Turning to economic theory for an answer one finds that one line of reasoning approaches the welfare costs of inflation in terms of what Bailey (1956) termed the "shoe leather" effect. The idea being that positive rates of inflation via an increase in the nominal rate of interest leads to a social inefficient low level of money demand which in turn causes excessive transactions costs. Arguing that money is socially costless to produce Friedman (1969) reasoned that inflation works as a tax, and proposed that the optimal rate of inflation is the one which makes the nominal interest rate equal to zero. With a positive real rate of return this would require a negative rate of inflation. However, putting the inflation tax in a 2nd best setting with other distortionary taxes (Phelps (1973)) in general implies that this tax should be used and thus inflation should be positive².

Another argument is that money is not in general super-neutral, that is, changes in the inflation rate affect via various mechanisms the allocation of resources (cf Tobin (1965)). While there is a strong theoretical case for the absence of super-neutrality, it is widely agreed that the quantitative importance is very modest. This is also reflected in numerous empirical studies attempting to identify a relationship between inflation and growth, but for moderate rates of inflation no such relation has been established, see eg Gylfason and Herbertsson (1998).

A third line of reasoning considers how inflation affects the efficiency of the price system does inflation contribute to improve or distort price signals? Two contradictionary views can be found in the literature. One is that inflation makes the price system function less efficiently since it leads to more variability in relative prices. Within a framework of incomplete

¹ For the European Central Bank the policy has been formulated as "price stability shall be defined as a year-on-year increase in the harmonised Index of Consumer Prices (HICP) for the euro area of below 2%" (European Central Bank(1999)). This is supplemented with the provision that this in practice means that inflation in the medium run should be in the interval between 0% and 2%.

 $^{^2}$ Feldstein (1996) argues that these effects are dominated by inflationary distortions in the taxation system, in particular between inflation and capital income taxation.

information in competitive markets this idea was extensively analysed as a side product to the New-classical macroeconomics (see eg Barro (1981)) for references and an introduction to this literature), and it was shown how excessive price variability could lead to misallocation of resources. Price variability or dispersion may also entail efficiency costs if it leads to excessive resource-absorbing search activities on the part of customers, cf Bénabou (1988,1992)³.

Another view is that inflation may work as grease for the price mechanism. The argument is that various sectoral shocks continuously require adjustment of relative prices, some have to increase and others to decrease. The adjustment process goes through nominal wage and price adjustment. At low rate of inflation some nominal prices (wage) will have to fall, and other to rise to ensure the proper adjustment of relative prices. At high rates of inflation, the required adjustment of relative prices may be ensured even if all nominal prices (wages) rise since some increase by more than others. However, at low rates of inflation the adjustment process may be strained to the extent that there are downward nominal rigidities.

This idea is informally developed in the seminal paper by Tobin (1972) in which he contends that:

Higher prices or faster inflation can diminish involuntary, disequilibrium unemployment, even though voluntary, equilibrium labour supply is entirely free of money illusion (Tobin, 1972, p 2).

Tobin's story of inflation as grease was based on a setting with continuous sectoral reallocations ("stochastic macroeconomic equilibrium") and thus a need for adjustment of relative prices. In this setting inflation may grease the adjustment process to the extent that there are downward nominal wage rigidities. While this idea has since been rather popular and often encountered in policy debates (see eg Edey (1995)), it has surprisingly not received much attention in the theoretical literature. One exception is the recent paper by Akerlof et al (1996)

³ Bénabou (1988,1992) has shown that this is indeed the case but also that one cannot draw any conclusions on the efficiency costs of inflation from this observation since the change in search activity has other implications. First, increased search activity tends to reduce the market power of firms and thereby the inefficiency loss caused by imperfect competition. Second, strategic complementarity in price setting implies that the optimal relative price is dependent on the rate of inflation although the direction is in general ambiguous. Hence, by introducing search behaviour explicitly in the formulation of demand it follows that inflation has both favourable and unfavourable effects on resource allocation, and it is not possible to make general statements on which dominates. Diamond (1993) develops a model in which the positive effects dominate for small rates of inflation and the negative dominates for high rates of inflation implying that there is an optimal non-negative rate of inflation.

which embodies the idea in a dynamic model.⁴

However, one important shortcoming of this argument is that it relies on exogenously imposed downward nominal rigidities, and the incentives of price setters are not considered. This is particularly problematic since models generating downward rigidities, tend to imply that downward rigidities are less prevalent at low rates of inflation (see e.g. Tsiddon(1991)).

The aim of this paper is to consider an alternative channel through which information may affect the efficiency of the price system. The incentive to change prices depends on changes in market conditions, that is, the realization of both real and nominal shocks. The higher the rate of inflation, other things being equal, the larger the incentive to change nominal prices to maintain the optimal relative price and vice versa. This suggests that relative prices are more flexible and therefore that the price mechanism works better as a resource allocation mechanism at higher rates of inflation. At very low rates of inflation the incentive to change prices is reduced and this may interfere with the need to change prices to attain an efficient resource allocation.

There is by now a rich variety of models generating nominal rigidities featuring adjustment costs, information problems, synchronization issues and strategic interactions, cf eg Andersen (1994) for an introduction). Since models based on adjustment costs (menu cost models) are the easiest to handle we shall use this as a workhorse to explore the implications of inflation for price adjustment and resource allocation. The specific case considered here involves nominal price adjustments but an argument along the same lines could be developed for nominal wage adjustment. Specifically, the model of monopolistic competition which has become the workhorse model of much modern macroeconomic analysis (see eg Romer (1996)) is used to address the issue of the trade off between the costs of adjusting prices and the incentive to adjust prices which in turn depends on real shocks and the underlying rate of inflation.

The paper is organized as follows: section 2 develops the theoretical model and considers the incentive to adjust prices for single firms and derives a measure of the misallocation of resources due to nominal price rigidities. Section 3 considers the optimal rate of inflation, section 4 reviews empirical evidence supporting that nominal rigidities and inflation are related. Section 5 provides some concluding remarks.

⁴ The structure of the model is, however, so complicated that analytical solutions are not available, and the authors take resort to simulations to illustrate that downward nominal rigidities may produce a non-linear Phillips-curve. See Andersen (2000) for a simple model.

2. Inflation and price adjustment

Consider an economy with a continuum of monopolistic firms producing differentiated nonstorable products. Firms indexed by i are uniformly distributed on the unit interval and have a measure of 1. The representative consumer has a demand function defined over the commodities given as

$$\mathbf{x}_{i} = \mathbf{f}\left(\mathbf{p}_{i}\right) \quad \mathbf{f'} < 0$$

where p_i is the relative price charged for commodity i defined as P_i/M using the money stock as the nominal anchor.⁵ We assume that the elasticity of demand is constant and equal to η . The firm produces subject to a production function

$$y_i = \frac{1}{k_i} l_i \qquad k_i > 0$$

where k_i is an exogenous productivity parameter and l_i is labour input. Real profit can be written

$$V = \left(\frac{P_i}{M} - k_i \frac{W}{M}\right) f\left(\frac{P_i}{M}\right)$$

Assuming that nominal wages are proportional to the money stock ((W/M) = constant normalized to unity for convenience) to rule out that the argument relies on an imposed nominal wage rigidity we find that real profit to firm i charging a nominal price P_i and having marginal real costs k_i can be written

$$V\left(\frac{P_{i}}{M}, k_{i}\right) = \left(\frac{P_{i}}{M} - k_{i}\right) f\left(\frac{P_{i}}{M}\right)$$

The optimal price for firm i is readily found to be

$$P_{i}^{*} = m k_{i} M$$
, $m \equiv \left(1 - \frac{1}{\eta}\right)^{-1} > 1$

where η is the elasticity of demand determining the mark-up m.

It is assumed that marginal real costs of production are given as

 $k_i = 1 + \varepsilon_i$

⁵ Effectively assuming that aggregate prices are proportional to the aggregate price index, that is, the Caplin and Spulber (1987) aggregation result is implicitly taken to hold. This also rules out strategic complementarity in price setting and therefore a possibility of multiple equilibria to the price adjustment game.

where ε_i has a density function, $h(\varepsilon_i)$, which is symmetric around zero and with support in [- λ,λ], $\lambda>0$. The current money supply is related to the past money supply as $M \equiv (1+\pi)\overline{M}$, ie π is the money growth rate which under the assumption made above equals the rate of inflation.

The initial nominal price of the firm is assumed to be $\overline{P}_i (\equiv m \overline{M})$, that is, the optimal price given the past money supply and assuming that the real shock is realized at its expected value. The question is now whether firms will change their nominal price after they know the realization of the real shock determining their marginal production costs. For this problem to be non-trivial it is assumed that there is a fixed real cost of adjusting the price (c).

The shock to the marginal costs of production is a real shock creating a need for adjustment of relative prices. This interacts with the need for nominal price adjustment created by inflation. The firm has thus to weight the cost of having an inoptimal nominal price relative to the cost of adjusting the price. Proceeding with the usual method adopted in the menu-cost literature, we use that the profit when charging the price P_i rather than the optimal price P^* up to a second order approximation can be written

$$V\left(\frac{P_{i}}{M}, k_{i}\right) = V\left(\frac{P_{i}^{*}}{M}, k_{i}\right) + V_{\frac{P_{i}}{M}}\left(\frac{P_{i}^{*}}{M}\right)\left(\frac{P_{i}}{M} - \frac{P_{i}^{*}}{M}\right)$$
$$+ \frac{1}{2}V_{\frac{P_{i}}{M}}\left(\frac{P_{i}^{*}}{M}\right)\left(\frac{P_{i}}{M} - \frac{P_{i}^{*}}{M}\right)^{2}$$

Since by definition $V_{\frac{P_i}{M}}\left(\frac{P_i^*}{M}\right) = 0$, it follows that the profit loss from maintaining the initial (now inoptimal) nominal price is given as

$$V\left(\frac{P_{i}^{*}}{M}, k_{i}\right) - V\left(\frac{\overline{P}_{i}}{M}, k_{i}\right) = \phi\left(\frac{\overline{P}_{i}}{M} - \frac{P_{i}^{*}}{M}\right)^{2}$$

where

$$\phi \equiv -\frac{1}{2} V_{\frac{P_i}{M}}^{"} \left(\frac{P_i^*}{M} \right) > 0$$

It follows straightforward that the firm decides not to change its price if

$$\left(\frac{\overline{P}_{i}}{M} - \frac{P_{i}^{*}}{M}\right)^{2} < \frac{c}{\varphi}$$

Using the approximation $(1 + x)^{-1} \cong 1 - x$, the condition that the firm keeps an unchanged price can be written

$$(\pi + \varepsilon_i)^2 < \frac{c}{\phi m^2} \equiv \tau$$

This implies that the firm maintains its initial nominal price if the shock to marginal cost lies in the interval

$$-(\pi+\tau) < \varepsilon_i < \tau - \pi$$

The bounds determining the interval of realizations for the real shock for which prices are unchanged depend on the costs of changing prices measured by τ and the rate of inflation (π)

Intuitively, the higher the cost τ , the wider the interval supporting unchanged prices. The interval supporting an unchanged price is only symmetric around zero in the case of an underlying nominal growth rate of zero (π =0). If nominal growth is positive, it will be negatively biased, that is, unchanged prices tend to prevail for firms experiencing low marginal costs (low ε_i) and they therefore tend to have a too high price (initial price relative to the new optimal price). This gives an asymmetry or downward rigidity such that the higher the rate of inflation, the more prices are downward rigid. The fact that the interval supporting fixed prices moves "leftwards" when inflation increases, means that the consequences of nominal price rigidities can be affected by changing the rate of inflation.

The probability that a given firm keeps a fixed price is given as

$$\Psi \equiv \int_{-(\pi+\tau)}^{\tau-\pi} h(\varepsilon_{i}) di$$

By assumption this is the fraction of firms not changing their price.

It is easily shown that inflation is inducing price flexibility since

$$\frac{\partial y}{\partial p} = -h(t-p) + h(-(t+p)) \le 0 \text{ for } p \ge 0$$

that is, the higher the rate of inflation, the smaller the fraction of firms keeping unchanged nominal prices. The intuition for this result is that if inflation is low, the need to adjust prices is lower and this produces nominal price rigidities. This implies not only that nominal shocks have real effects, but also that the adjustment to real shocks (here the ε_i 's) is impaired.

Considering how price rigidity affects relative prices we have

$$\frac{\mathbf{P}_{i} / \mathbf{M}}{\mathbf{P}_{i}^{*} / \mathbf{M}} = \frac{1}{(1 + \pi)(1 + \varepsilon_{i})}$$

It follows that inflation tends to imply that the firms who do not adjust their price tend to have a "too low" price, while the real shock implies that firms with high (low) marginal costs tend to have a too low (high) price.

Turning to aggregate activity we find a non-linear relationship between aggregate activity and inflation. To see this note that aggregate activity can be written

$$\begin{aligned} \mathbf{x} &= \int \mathbf{f}\left(\frac{\mathbf{P}_{i}}{\mathbf{M}}\right) \mathbf{h}(\boldsymbol{\varepsilon}_{i}) d\mathbf{i} \\ &= \int_{-\boldsymbol{\lambda}}^{-(\boldsymbol{\pi}+\boldsymbol{\tau})} \mathbf{f}(\mathbf{k}_{i}\mathbf{m}) \mathbf{h}(\boldsymbol{\varepsilon}_{i}) d\mathbf{i} + \int_{-(\boldsymbol{\pi}+\boldsymbol{\tau})}^{\boldsymbol{\tau}-\boldsymbol{\pi}} \mathbf{f}(\mathbf{m}(\mathbf{1}+\boldsymbol{\pi})^{-1}) \mathbf{h}(\boldsymbol{\varepsilon}_{i}) d\mathbf{i} + \int_{\boldsymbol{\tau}-\boldsymbol{\pi}}^{\boldsymbol{\lambda}} \mathbf{f}(\mathbf{k}_{i}\mathbf{m}) \mathbf{h}(\boldsymbol{\varepsilon}_{i}) d\mathbf{i} \end{aligned}$$

and it follows that

$$\frac{\partial x}{\partial \pi} \geq 0$$

and

$$\frac{\partial \mathbf{x}}{\partial \boldsymbol{\pi}} \to 0 \text{ for } \boldsymbol{\pi} \to \infty$$

There is a non-linear relationship(Phillips curve) between aggregate activity and the rate of inflation. Higher inflation leads to higher activity, but eventually the effect dies out. This is similar to the findings of Tobin (1972) and Akerlof et.al. (1986) except that it is here derived in a setting where price adjustment is made endogenous.

3. The Optimal Rate of Inflation

The preceding analysis suggests that the choice of the rate of inflation may affect the efficiency of the price system. High inflation necessitates price adjustments which are good for resource allocation, but entails adjustment costs. On the other hand, low inflation implies price rigidities,

and low adjustment costs, but an inadequate adjustment of prices. This suggests a trade-off between the costs of price changes and the efficiency of the price system in allocating resources.

A welfare evaluation is complicated in the present setting by the market imperfections caused by the market power of price-setting firms. This does in itself create a welfare loss relative to the case of perfect competition. This is reflected in the mark-up parameter (m). Since this is unaffected by inflation, we use the case of fully flexible prices as the benchmark for the welfare considerations. To assess the welfare consequences of inflation, we thus adopt a 2nd best approach in the sense that market imperfections imbedded in the market power of firms are taken as given⁶. It is thus asked what rate of inflation maximizes the welfare of consumers taken as given the market power of firms (constrained efficiency), see appendix.

The misallocation of resources caused by price rigidity can be written (see appendix)

$$A(\pi) = \chi \int_{-(\tau+\pi)}^{\tau-\pi} (\pi+\varepsilon_i)^2 h(\varepsilon_i) d\varepsilon_i, \chi > 0$$

We have that

$$\frac{\partial A(\pi)}{\partial \pi} = \chi \tau^2 \Big[h \big(-(\tau + \pi) \big) - h(\tau - \pi) \Big]$$

$$+\chi \int_{-(\tau+\pi)}^{\tau-\pi} (\pi+\varepsilon_{i})h(\varepsilon_{i})d\varepsilon_{i} \geq 0$$

The effects of inflation on the misallocation of resources is thus in general ambiguous, since two effects are present. First, higher inflation reduces the fraction of firms keeping a rigid price by moving the interval supporting rigid prices leftward. Second, the higher the inflation rate, the larger the discrepancy between the optimal and the rigid price and thus resource misallocation for those firms not changing their prices. It is therefore not possible to make an unambiguous statement concerning the relationship between the efficiency of the price system and the rate of inflation.

⁶ This disregards the fact that money needs not be super-neutral, that is, changes in the rate of inflation may change the optimal relative price, production level for imperfectly competitive firms (see Naish(1986), Kuran(1986a,b),Konieczny(1990)). Danziger(1988) presents an interesting model in which an increase in the rate of inflation effectively reduces the market power of firms and thereby becomes welfare-improving. Notice, that this is a completely different argument than the one discussed here in relation to adjustment to shocks.

However, a zero inflation rate is in terms of allocation efficiency dominated by a strictly positive rate of inflation ($\hat{\pi} > 0$), i.e. (see appendix for proof)

$$A(\hat{\pi}) < A(0)$$
 for $\hat{\pi} > 0$

that is, there are efficiency gains in terms of a better allocation of resources to be reaped by choosing a strictly positive rate of inflation.

Although there is not a monotone relation between inflation and misallocation of resources due to the two counteracting effects mentioned above, it can be shown (see appendix) that

$$\lim_{\pi\to\infty} A(\pi) = 0$$

ie at very high rates of inflation nominal prices are fully flexible and hence price rigidity is not a source of misallocation of resources.

From this one may infer that a very high rate of inflation is optimal since it eliminates the distortions caused by nominal rigidities. Such a conclusion disregards the costs of inflation. In the present model this is captured by the cost associated with price adjustment and the optimal rate of inflation is found as the solution to

 $\min_{\pi} A(\pi) + (1 - \Psi(\pi))c$

where the first term is the misallocation term and the second is the costs of price adjustment (= fraction of firms changing prices times the adjustment cost).

While it is difficult to characterize the optimal rate of inflation in general, it is possible to conclude that it is strictly positive (see appendix). The intuition why there is a welfare case for having a positive rate of inflation is that low inflation strengthens nominal rigidities. If inflation is low (zero) firms are less concerned about adjusting prices but this interferes with the need for price adjustment to cope with real shocks so as to ensure an efficient resource allocation. In short inflation may work as grease by affecting the incentives for price adjustment to imply more price flexibility and thereby improve efficiency in allocation.

Finally, note that the argument made here is that inflation interacts with the incentives underlying price adjustment so as to have implications for resource allocation despite the fact that monetary neutrality prevails in the long run. The present argument is not exploiting the non-neutrality of money implied by nominal price rigidities, that is, the gain included in the comparison made here does not include the short run gain in activity which follows from a (unanticipated) monetary expansion in the presence of nominal rigidities. The reason being that the comparisons have been made relative to the allocation under imperfect competition with

flexible prices.

4. Empirical evidence

The essential argument in the preceding analysis is that nominal price or wages are more rigid especially in a downward direction at low rates of inflation. It is therefore natural to question whether there is any empirical evidence in support of this mechanism.

There is a voluminous literature employing various approaches to identify nominal rigidities. It is beyond the scope of the present paper to review this literature and attention is restricted to studies using micro data, since they are most relevant for the present discussion. Considering the distribution of nominal wage changes it is possible to identify nominal rigidities by spikes at zero or asymmetries in the distribution of wage changes. The recent literature following this approach starts with MacLaughlin(1994) who finds little evidence in support of nominal rigidities for US data, a similar conclusion in reached on UK data by Smith(2000). However, a long list of other studies along this line have found support for nominal ridigidies. Most studies are on US data and the list includes Kahn(1997), Card and Hyshlop(1997), Akerlof, Dickens and Perry (1996), Groshen and Schweitzer (1999), and Altonji and Devereux(1999). Similar conclusions on Canandian data are reported by Christofides and Leung (1999), on German data by Beissinger and Knoppik(2000), and on Uk data by Yates(1999).

More interesting is the fact that by considering this problem across periods with different levels of inflation one finds that the prevalence of nominal rigidities is stronger in low than in high inflation periods. For US data this is reported by Card and Hyshop(1997), on Canadian data by Christofides and Leung(1999). Based on an analysis of survey data for Sweden Agell and Lundborg (1999) also find that low inflation strengthens nominal rigidities.

Related to the results reported above is the issue of contract length, since longer duration of nominal contracts would obviously strengthen nominal rigidities. Empirical evidence also confirms that the contract length is decreasing in the rate of inflation, see.e.g Vroman (1989) and Murphy(1992). Rich and Tracy(2000) find that contract length is unambiguously decreasing in inflationary uncertainty, and since the latter is usually found to be increasing in the rate of inflation, this finding also suggests that nominal rigidities are more prevalent in periods of low inflation

Finally, it is worth pointing out that another implication of the theoretical analysis is that inflation may contribute to less relative price variability by inducing price adjustment. This runs

counter to the usual perception that inflation contributes to price variability. It is noteworthy that empirical support to the assertion that a higher rate of inflation is associated with increased price dispersion is very weak, and that there might in fact be some indication for the reverse association⁷. See Bryan and Cecchetti (1996) for evidence on US data and Fielding and Mizen(2000) for evidence for European countries.

5. Concluding Remarks

This paper has shown that inflation may enhance the efficiency of the price system. The key to this result is to explicitly recognise the incentives of price setter to change prices. If various mechanisms like adjustment costs impair nominal price adjustment, it follows that relative prices may adjust insufficiently to the shocks impinging on markets. However, the rate of inflation also affects the incentive to change prices, and a positive rate of inflation contributes to more price flexibility and therefore a more efficiently working price system.

While the present analysis has taken a normative approach, the finding may also be relevant for positive theories of inflation. The well-known time-inconsistency arguments (cf. Kydland and Prescott (1977) and Barro and Gordon(1983)) has that inflation is driven by the effects unanticipated inflation has on activity. The present analysis shows that (anticipated) inflation also play a role for the efficiency of the price system and therefore for aggregate activity.

Since the argument presented here is developed in a static model it is natural to question whether the results carries over to an explicit dynamic setting. The important point of the present analysis is that inflation induces price adjustment, and in the presence of real shocks necessitating price adjustment it follows that more flexibility in price setting may be beneficial. Explicit dynamic menu cost models also support that inflation is conducive to price adjustment (see the survey in Andersen (1994)). Hence, it is reasonable to conjecture that the basic mechanism analysed here carries over to an explicit dynamic setting.

The argument that inflation can grease the price mechanism has recently been dismissed by King(1999)

"...it is difficult to believe that any downward inflexibility of nominal wages would be unaffected by changes in inflation. As low inflation becomes the norm, resistance to nominal wage cuts could well disappear" (King(1999, page 18)).

⁷ Note that even though inflation variability may be positively related to inflation, this is not relevant for the present discussion about optimal rate about which inflation should be stabilized.

The present explicit analysis of the incentive to change nominal prices (or wages) has shown that the relation may be the opposite, that is, with low inflation nominal rigidities are reinforced.

Quantitative studies matching knowledge about nominal wage and price adjustment with the occurrence of sector specific shocks are very scant. Accordingly it is difficult to make reliable quantitative assessments of the optimal rate of inflation. However, the gist of the argument suggests that this is a relatively small number, and that adjustment processes work better at an inflation rate of say 2 - 3 percent per year as compared to a zero rate of inflation.

Quantitative assessments of the optimal rate of inflation have recently been made in which the "Shoe-leather" effects of inflation (Lucas (1995)) and the distortions arising from imperfect indexation of the tax system (Feldstein (1996)). It is found that there can be considerable welfare gains in moving to price stability. None of these studies do, however, attempt to include the allocational implications of low inflation in the presence of nominal rigidities. An attempt to quantify the effects hereof is made in Akerlof et al (1996) assuming downward nominal rigidities. They calibrate their model to the US economy and find that a movement from 3 percent inflation to price stability would imply an increase in the unemployment rate of at least 1 percentage point. It is still an open question what an analysis including all these aspects of inflation would imply with respect to the optimal rate of inflation.

Appendix: Welfare Effects of Resource Misallocation

Note that output and thus consumption of good i can be written

$$\mathbf{y}_{i} = \mathbf{f} \left(\mathbf{P}_{i} / \mathbf{M} \right)$$

Hence, utility from consumption of this good is given as

$$u(y_i) = u(f(P_i / M)) = z(P_i / M); z' < 0, z'' < 0$$

To evaluate the indirect utility at relative prices different from the optimal relative prices, we use the approximation

$$z\left(\frac{P_{i}}{M}\right) = z\left(\frac{P_{i}^{*}}{M}\right) + z'_{\frac{P_{i}}{M}}\left(\frac{P_{i}^{*}}{M}\right)\left(\frac{P_{i}}{M} - \frac{P_{i}^{*}}{M}\right)$$

$$+\frac{1}{2} z_{\frac{P_i}{M}}^{"} \left(\frac{P_i^*}{M}\right) \left(\frac{P_i}{M} - \frac{P_i^*}{M}\right)^2$$

Assuming a utilitarian welfare criterion and taking the market imperfections imbedded in the market power of firms for given, (ie constrained optimality) implies that the optimum is defined for relative prices, implying

$$z'_{\frac{P_i}{M}}\left(\frac{P_i^*}{M}\right) = 0$$

It follows by using a standard 2nd order Taylor approximation that

$$z\left(\frac{P_{i}}{M}\right) - z\left(\frac{P_{i}^{*}}{M}\right) = \frac{1}{2} z \frac{\mu_{i}}{M} \left(\frac{P_{i}^{*}}{M}\right) \left(\frac{P_{i}}{M} - \frac{P_{i}^{*}}{M}\right)^{2}$$

Inserting the price formula, we find

$$\frac{1}{2} z_{\frac{P_i}{M}}^{"} \left(\frac{P_i^*}{M_i}\right) \left(\frac{P_i}{M} - \frac{P_i^*}{M}\right)^2 = \frac{1}{2} z_{\frac{P_i}{M}}^{"} \left(\frac{P_i^*}{M}\right) \left(\frac{m}{1+\pi} - m\left(1+\varepsilon_i\right)\right)^2$$

Using the approximation $\frac{1}{1+x} \sim 1-x$ for x small and defining

$$\chi \equiv -\frac{1}{2} z \frac{\tilde{P}_{i}}{M} \left(\frac{P_{i}^{*}}{M} \right) m > 0$$

we get that the welfare costs of the misallocation of good i induced by price rigidity can be written

$$\chi(\pi+\epsilon_i)^2 \ge 0$$

Aggregating we find the total costs of misallocation to be

$$A(\pi) \equiv \chi \int_{-(\tau-\pi)}^{\tau-\pi} (\pi+\varepsilon_i)^2 h(\varepsilon_i) d\varepsilon_i$$

it is assumed that

$$A(0) = \chi \int_{-\tau}^{\tau} (\varepsilon_i)^2 h(\varepsilon_i) d\varepsilon_i > c$$

that is, there is a trade-off between price adjustment and costs of adjustment.

It follows that

$$\frac{\partial A}{\partial \pi} = 2\chi \int_{-(\tau+\pi)}^{\tau-\pi} (\pi+\varepsilon_i) h(\varepsilon_i) d\varepsilon_i - \chi \tau^2 [h(\tau-\pi) - h(-(\tau+\pi))]$$
$$\frac{\partial^2 A}{\partial \pi^2} = 2\chi \int_{-(\tau+\pi)}^{\tau-\pi} h(\varepsilon_i) d\varepsilon_i - 2\chi [h(\tau-\pi) - h(-(\tau-\pi))]$$
$$-\chi \tau^2 [-h'(\tau-\pi) + h'(-(\tau-\pi))]$$

It is easily verified that there exists a $\pi > 0$ for which $A(\pi) < A(0)$ by using that

$$\frac{\partial A}{\partial \pi}\Big|_{\pi=0} = 0$$
$$\frac{\partial^2 A}{\partial \pi^2}\Big|_{\pi=0} < 0$$

Note also that

 $A(\pi) \rightarrow 0$ for $\pi \rightarrow \infty$

Consider finally the problem of choosing the rate of inflation $\pi \ge 0$ which solves

$$\min_{\pi} A(\pi) + (1 - \psi(\pi))c$$

The first order condition to this problem is

$$\frac{\partial A}{\partial \pi} = c \frac{\partial \psi(\pi)}{\partial \pi}$$

and the second order condition

$$\frac{\partial^2 A(\boldsymbol{\pi})}{\partial \boldsymbol{\pi}^2} - c \frac{\partial \boldsymbol{\psi}^2(\boldsymbol{\pi})}{\partial \boldsymbol{\pi}^2} > 0$$

We shall prove that if this problem has a solution the optimal rate of inflation is strictly positive. The proof runs by contradiction. Assume that the optimal solution is $\pi=0$. We have that the first order condition is fulfilled for $\pi=0$, since

$$\frac{\partial A(\pi)}{\partial \pi} = \frac{\partial \psi(\pi)}{\partial \pi} = 0 \text{ for } \pi = 0$$

but the second order condition is not since

$$\frac{\partial^2 \mathbf{A}}{\partial \boldsymbol{\pi}^2} \Big|_{\boldsymbol{\pi}=0} < 0$$

and

$$\frac{\partial^2 \boldsymbol{\Psi}}{\partial \boldsymbol{\pi}^2} \Big|_{\boldsymbol{\pi}=0} = 0$$

Hence, a contradiction, and it can be concluded that the optimal rate of inflation is stricly positive.

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