

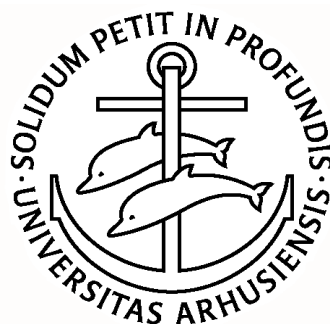
# DEPARTMENT OF ECONOMICS

## Working Paper

STRUCTURALLY DEPENDENT COMPETING RISKS

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# Structurally Dependent Competing Risks\*

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## Abstract

In this paper, we specify and estimate a structurally dependent competing risks model for the transitions out of unemployment into either new job or recall. The recall probability is allowed to affect the search intensity for new jobs.

**JEL Classification:** C41, J64

**Keywords:** Competing risks , structural dependence, recall hazard, new job hazard.

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# 1 Introduction

In this paper we specify and estimate a simultaneous equations model for hazards. The model proves useful in applications where the processes describing different dynamic situations are structurally dependent. Our particular example considers the process for unemployment durations. We consider unemployed workers searching for jobs. Job offers are assumed to arrive from either the previous employer (recall) or from a new employer. Recalls may carry a higher wage due to accumulation of firm-specific human capital. This implies that the intensity of search for new jobs will be inversely related to the recall probability<sup>1</sup>. Hence the term structural dependence.

The paper contributes to two different branches of the literature. First, the application adds a new dimension to the distinction between recall versus new job transitions out of unemployment. Katz (1986) estimated the two transitions in a competing risks model assuming that the two hazards are independent, conditional on some observed variables. Han & Hausman (1990) extended the analysis by allowing for dependence between the recall hazard and the new job hazard through correlation between unobserved variables. In this paper we extend the Han & Hausman model by incorporating structural dependence between the hazard rates. We do this by including the recall hazard as an additional explanatory variable in the new job hazard.

Second, the model provides an alternative specification of a simultaneous hazard model, compared to the models developed by Lillard (1993) or Fallick & Ryu (1997).

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<sup>1</sup>See Fallick & Ryu (1997) for a theoretical model that implies this relationship between the recall hazard and the new job hazard.

Lillard’s model assumes that the unobserved variables are jointly normally distributed, and he obtains identification of the structural dependence parameter by exclusion restrictions on the covariates. Fallick & Ryu rely on identification by functional form nonlinearities. In our model, we show that identification may be obtained without exclusion restrictions, and without resorting to functional form assumptions, when repeated spells are available.

The paper is organized as follows. Section 2 presents the econometric model, and in Section 3 we describe the data. Section 4 contains the results, and in Section 5 we conclude.

## 2 Econometric Model

The econometric model is a competing risks hazard model<sup>2</sup>. Each of the destination specific hazards is specified as a mixed proportional (sub-) hazard. The destination specific hazard for recall is

$$h_r(t|x, v_r) = \lambda_r(t) \exp(\beta'_r x + v_r). \quad (1)$$

where  $x$  is observed and  $v_r$  is unobserved. The destination specific hazard rate for new job is specified in the following way

$$h_n(t|x, \varepsilon_n) = \lambda_n(t) \exp(\beta'_n x + v_n + \alpha h_r(t|x, v_r)) \quad (2)$$

Note, that the parameter  $\alpha$  captures the structural dependence of the new job hazard rate on the recall probability. Fallick & Ryu (1997) only included the recall baseline

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<sup>2</sup>See Lancaster (1990) for more on hazard models.

hazard ( $\lambda_r(t)$ ) in the new job hazard (thus leaving out the individual specific part of the recall hazard). This implies that, in their case, identification was obtained through functional form assumptions. They found strong effects in the patterns of duration dependence from introducing the structural element into the model. However, since there is no variation in the recall baseline hazard between persons, their specification amounts to being a highly nonlinear respecification of the new job baseline hazard rate, that is, instead of  $\lambda_n(t)$ , the new job baseline becomes  $\lambda_n(t) \cdot \exp(\alpha \lambda_r(t))$ . Lillard (1993), on the other hand, included the entire recall hazard in the new job hazard and relied on exclusion restrictions for identification.<sup>3</sup>

In our case, there are various sources of variation providing identification. The parameters of the recall hazard is identified from data on recalls alone. The parameters of the new job hazard are identified, and  $\alpha$  is identified through three sources of variation, namely variation in the recall hazard rate over time (the baseline), variation in the level of the recall hazard between individuals (observed and unobserved variables), and variation in the recall hazard within individuals across spells (observed variables), as an individual may be observed in more than one spell of unemployment (see section 3).

In deriving the likelihood function, we first specify some distributional assumptions and introduce some notation: Assume that the baseline hazards are piecewise constant with splitting times  $\tau_0, \tau_1, \dots, \tau_K$ , with  $\tau_0 = 0$ ,  $\tau_K = +\infty$ . Let the values of the baseline hazard rates in the  $k$ 'th interval be given by  $\lambda_r^k, \lambda_n^k$ . Define the mapping

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<sup>3</sup>The principle of exclusion restrictions is well suited for Lillard's application (joint distribution of the duration of marriage and timing of marital conception), but not for Fallick & Ryu's or ours.

$k(t) : \mathbb{R}_+ \curvearrowright \{1, 2, \dots, K-1, K\}$ , which maps a duration,  $t$ , into an interval,  $k$ . Finally, the destination state  $J = n, r$  gives two destination indicators,  $d_n = \mathbb{I}_{\{J=n\}}$ ,  $d_r = \mathbb{I}_{\{J=r\}}$ . The likelihood contribution for a single unemployment spell, conditional on observed and unobserved variables, may now be expressed as

$$\mathcal{L}(\theta) = h_r(t|x, v_r)^{d_r} \cdot h_n(t|x, v_n, v_r)^{d_n} \cdot \exp \left[ - \int_0^t h_r(s|x, v_r) ds - \int_0^t h_n(s|x, v_n, v_r) ds \right]$$

The unobserved variables are assumed to be individual specific (that is, the random effect is assumed to be constant across different spells for the same individual), and to follow a discrete distribution with  $2 \times 2$  points of support. Since there is already a constant (i.e. the baseline hazard) in each of the destination specific hazards, we make the normalization that one of the support point in each destination specific hazard takes the value zero.

### 3 Data

The data is a flow sample of all unemployment spells initiated by a 0.2% sample of the Danish population during the period Jan. 1, 1981 to Dec. 31, 1990. The data is extracted from registers used for UI-benefit payments. For each unemployment spell, the duration of the spell and the subsequent destination (recall, new job, or something else) is known. If the spell does not end before Dec. 31, 1990, it is treated as independent right censoring. The same holds for transitions out of the labour force.

For the purpose of the present analysis, we select men in the age group 25-59. The

sample contains 1422 individuals with at least one fresh unemployment spell during the period, and a total of 7781 fresh unemployment spells (5.5 spells on average per person), of which 3950 end with a transition into a new job, and 3016 end with a recall.

A number of explanatory variables are used in the estimations; Age, actual working experience, educational level, demographic and geographic variables as well as UI-fund membership indicators. Their coefficients are, however, not reported below, due to limitations of space.

## 4 Estimation Results

The parameter estimates for the coefficients of the model are shown in Table 1. In Model A we estimate the model without introducing structural dependence (that is, the traditional competing risks model), whereas in Model B, we have allowed for structural dependence in the manner of Fallick and Ryu, by including the recall baseline as an explanatory variable in the new job hazard.<sup>4</sup> Finally, Model C presents the estimation results for the specification presented above.

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<sup>4</sup>Model B still differs of that by Fallick and Ryu in that we exploit the availability of repeated spells to identify the unobservables' distribution.



TABLE 1: ESTIMATION RESULTS (Asymptotic standard errors in italics)

	Model A		Model B		Model C	
$v_{2,r}$	1.6968	<i>0.0491</i>	1.7673	<i>0.0542</i>	1.7179	<i>0.0381</i>
$v_{2,n}$	1.2112	<i>0.0471</i>	1.2156	<i>0.0479</i>	1.2879	<i>0.0448</i>
$\Pr(v_{1,r}, v_{1,n})$	0.4901	<i>0.0283</i>	0.4649	<i>0.0275</i>	0.5522	<i>0.0229</i>
$\Pr(v_{1,r}, v_{2,n})$	0.2847	<i>0.0222</i>	0.3171	<i>0.0239</i>	0.2579	<i>0.0234</i>
$\Pr(v_{2,r}, v_{1,n})$	0.1694	<i>0.0247</i>	0.1505	<i>0.0230</i>	0.1001	<i>0.0156</i>
$\Pr(v_{2,r}, v_{2,n})$	0.0558	<i>0.0158</i>	0.0675	<i>0.0177</i>	0.0898	<i>0.0181</i>
$\alpha$			0.0996	<i>1.0284</i>	-2.8604	<i>0.4867</i>
$Corr(v_r, v_n)$	-0.1055	<i>0.0571</i>	-0.0815	<i>0.0589</i>	0.1271	<i>0.0674</i>
$\text{Log}\mathcal{L}$	-26,063		-26,061		-26,052	

The (unreported) effect from the covariates on the three hazard rates are not strongly affected by the inclusion of the recall hazard in the new job hazard.<sup>5</sup>

The distribution of the unobserved heterogeneity terms is quite similar between the three specifications. There is, nevertheless one important difference. If we calculate the correlation of  $v_r, v_n$ , it is negative in Models A and B, albeit only significantly so in Model A. A negative correlation implies that individuals who, based on the unobserved components, are more prone to leave unemployment through recall, are less prone to leave unemployment for a new job. This result appears counter-intuitive. In Model C, the correlation is significantly positive, and the negative correlation of Model A (and B) is thus a consequence of neglecting the structural dependency that exists between the two hazards.

In Model B we find that the coefficient of structural dependence,  $\alpha$ , is very poorly determined due to the reliance on functional form identification only. In Model C we find a significant negative coefficient of structural dependence, which confirms the prediction that the intensity devoted to search for a new job declines in the recall probability. Our

<sup>5</sup>Full estimation results are available on request.

Model C does thus in all respects live up to our a priori expectations, in contrast with the Models A and B. In addition, the improvement of the likelihood function in going from either A or B to C is large.

In Figure 1 we depict the hazard functions. The recall hazard still exhibits negative duration dependence as expected, and the inclusion of the structural dependency does not alter the baseline profiles, except for a small change in the new job hazard, corresponding to the effect from the time-varying recall probability.

## 5 Conclusion

In this paper we specify and estimate a simultaneous equations model for hazards. Our particular example considers the process for unemployment durations, where the unemployed workers can leave unemployment for a job at their previous employer (recall) or for a job with a new employer. We argue that there is a structural dependency between the destination states, since individuals would prefer to return to their previous employer, and consequently the intensity devoted to search for a new job declines in the recall probability.

Our extension of the standard competing risks model appears fruitful since we find significantly different results when including the structural dependency. Specifically, we confirm the theoretical prediction that the recall hazard affects the new job hazard negatively, and that this phenomenon, if not accounted for by the statistical model, affects the correlation structure of the unobserved heterogeneity components. Our model

thus outperforms the standard competing risks model and the extension suggested by Fallick and Ryu (1997).

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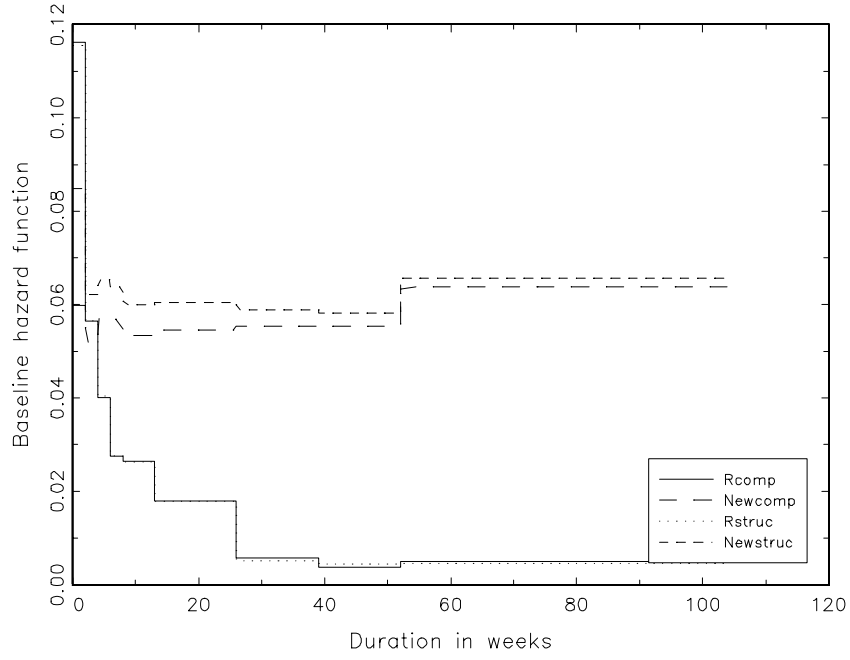


Figure 1: Baseline hazard function for Model A and Model C.

Note: Rcomp is the average baseline recall hazard for Model A, Rstruc is the average baseline recall hazard for Model C, Newcomp is the average new job baseline hazard for Model A, while Newstruc is the average new job baseline hazard for Model C.

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